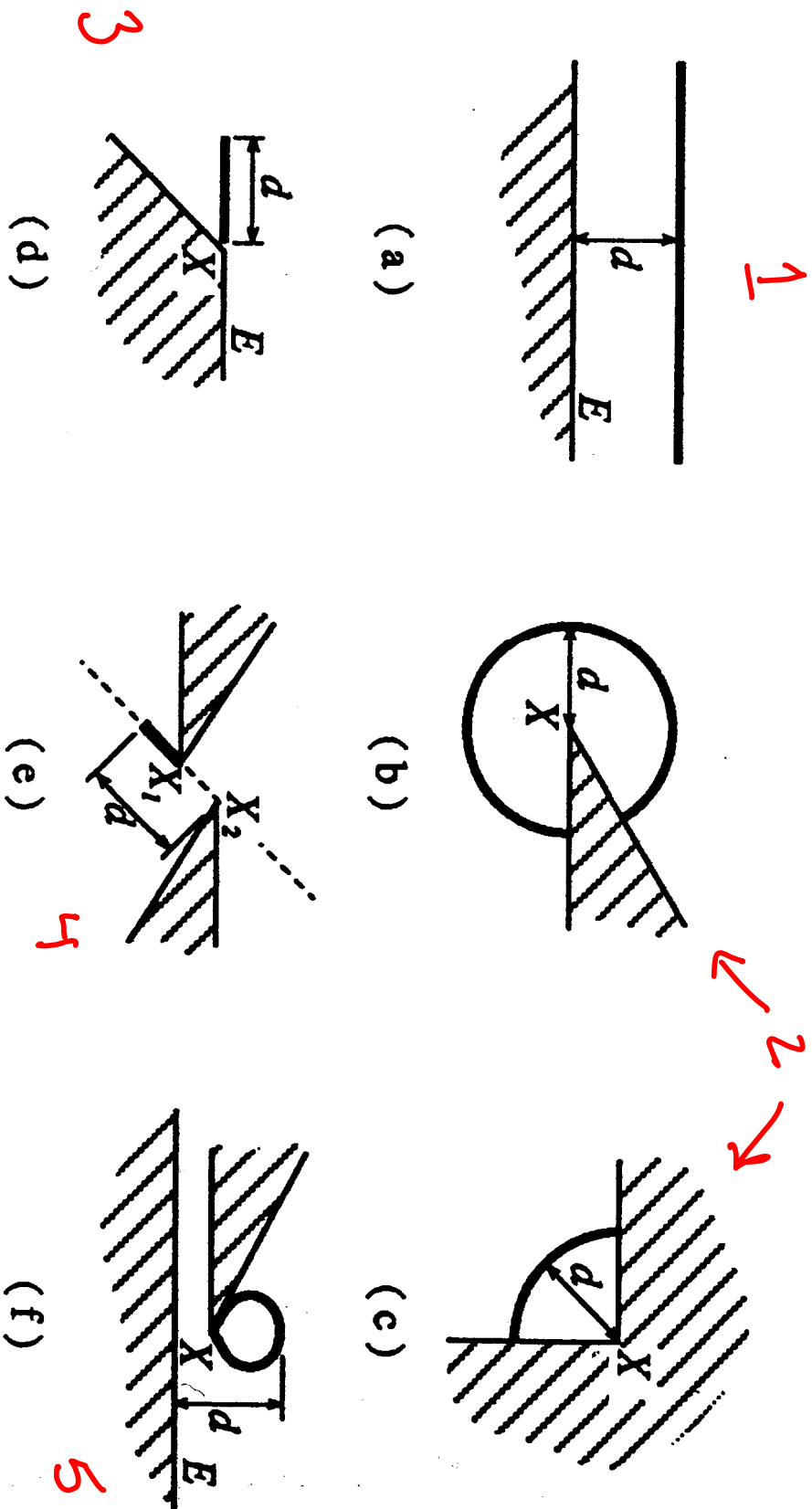


**Figure 12.** This figure shows the decomposition of a polygonal workspace into 13 noncritical regions. The robot can move from one end of the “corner” to the other but it cannot make a full rotation in the corner. Thus, when

**KEY IDEA:**  $\rightarrow$  project the "faces"  
of  $C$ -obstacles onto  $x$ - $y$  plane.  
 $\Rightarrow$  Critical curves in  $x$ - $y$  plane

Critical curves:

- 1) Boundaries of faces of  $C_{ob}$
- 2) Curves in  $C_{ob}$ 's faces where  $f$  tangent plane to the face is  $\perp$   $x$ - $y$  plane  
"low deg. polynomials in  $x$ - $y$ "  $\leq 4$



**Figure 5.** This figure illustrates the various types of **critical curves** other than the obstacle edges. The critical curves (shown in bold lines) are the set of **positions of  $A$  where the structure of the C-obstacle region along the  $\theta$  direction undergoes a qualitative change.**



$\beta_1 - \beta_6 \rightarrow \text{Type 1}$     $\beta_7 - \beta_{10} : \text{type 2}$

$\beta_{11} - \beta_{16} \rightarrow \text{Type 3}$     $\beta_{17} - \beta_{18} \rightarrow \text{type 4}$

$\beta_{19} - \beta_{21} \rightarrow \text{Type 5}$

Chapter 5: Exact Cell Decomposition

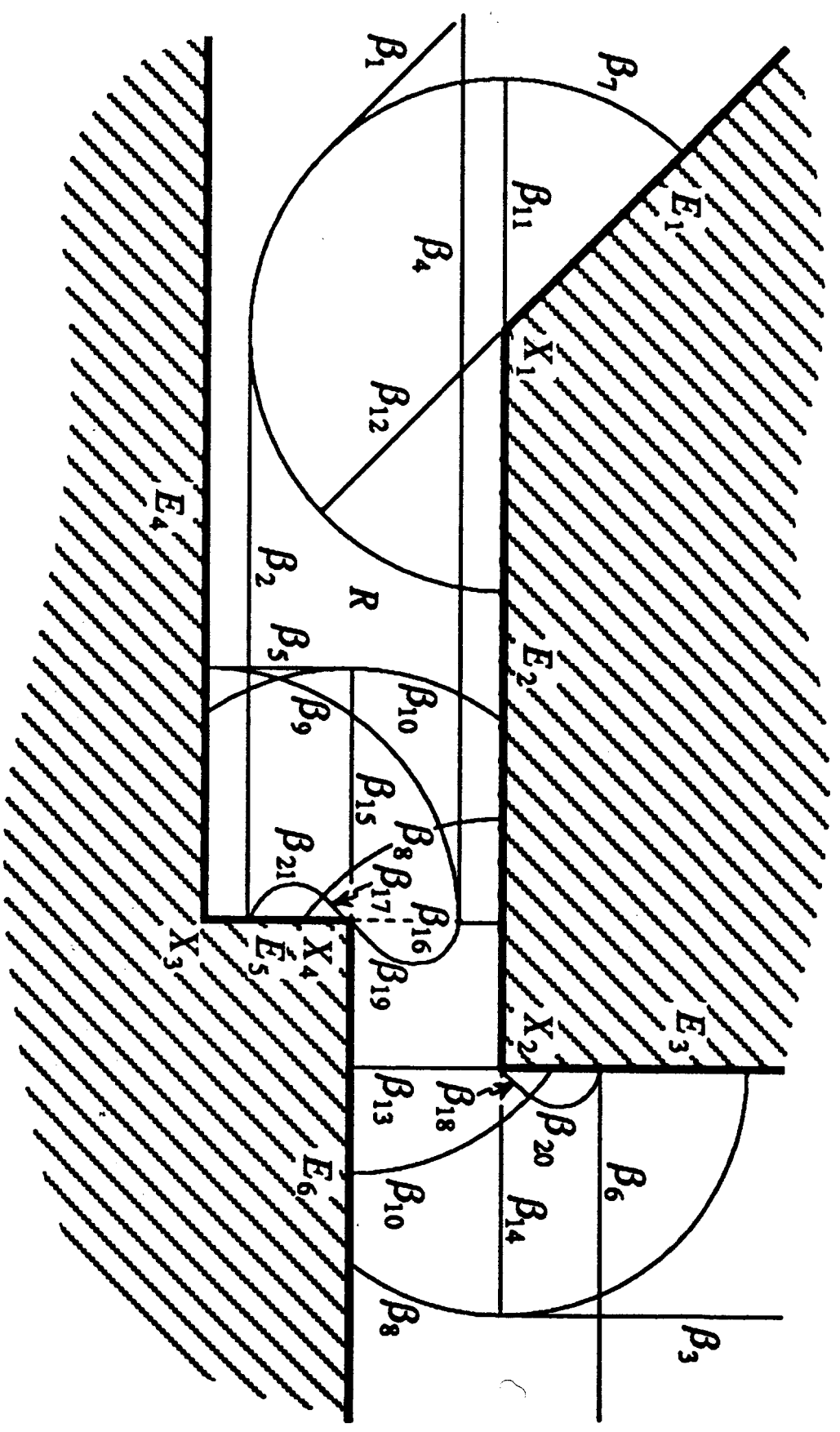


Figure 7. This figure illustrates the concepts of a critical curve and a non-

# Isolation and Rotation in the Plane

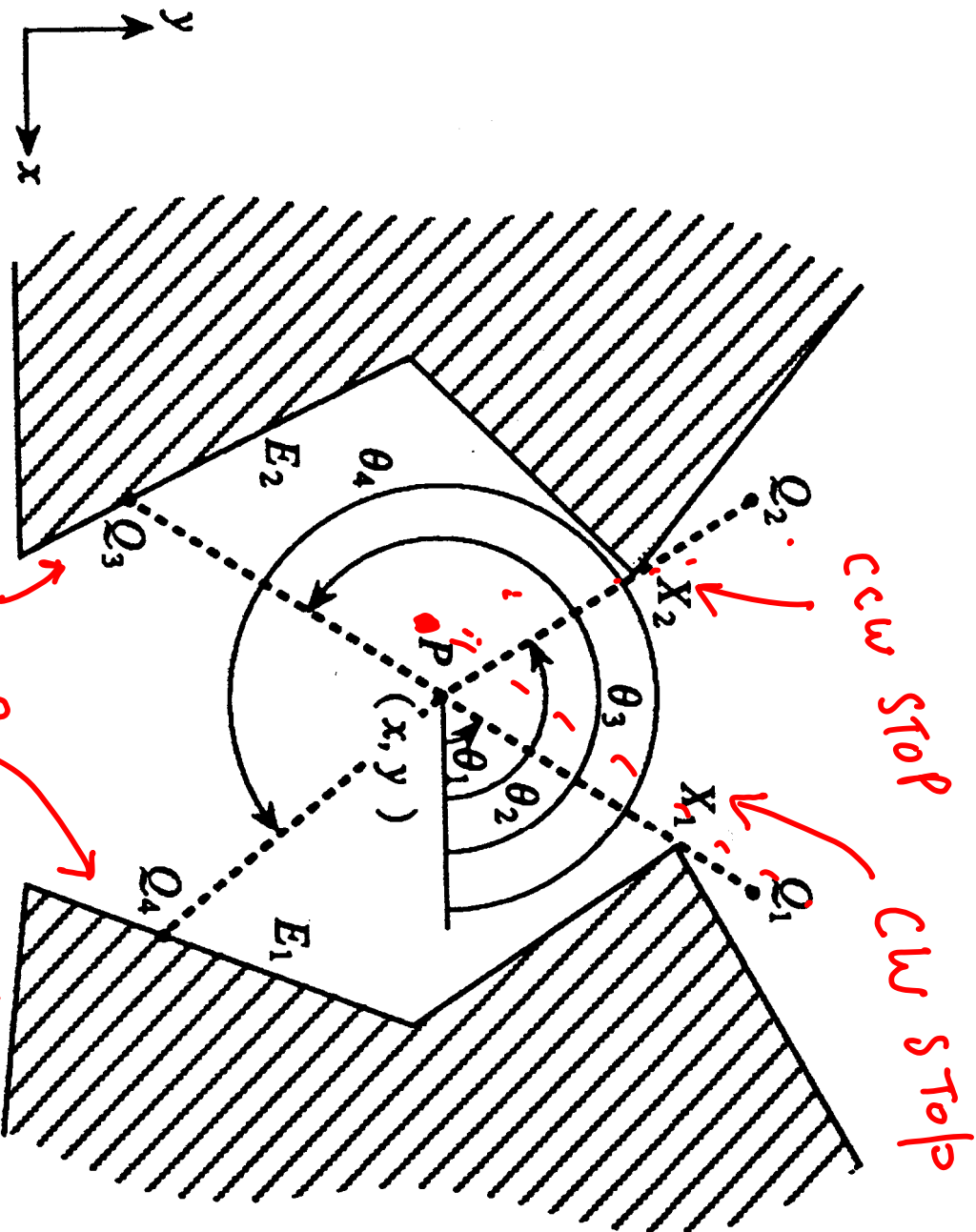


Figure 10. A is at a noncritical position  $(x, y)$ . The obstacle edges and vertices can touch without intersecting the interior of B when it rotates about a called stops. A stop is (counter)clockwise if it can be reached from a

Decomposition of free-space  
215

$$F(x, y) = \{ \theta : (x, y, \theta) \in C_{free} \}$$

= set of intervals

$\sigma(x, y) \downarrow$  corr. to 0.

$\downarrow$   $\{ [x_1, x_2], [E_1, E_2] \}$  stop

$\downarrow$   $\{ [x_1, x_2], [E_1, E_2] \}$  stop

$\downarrow$   $\{ [x_1, x_2], [E_1, E_2] \}$  stop

$$\lambda_1(x, y, x_1) = \theta_1 \quad \text{orient. of wires}$$

$$\lambda_2(x, y, x_2) = \theta_2 \quad \text{for shares } x_1$$

$$\text{Cell} = f(x, y, \theta) \quad ; \quad (x, y) \text{ share } \checkmark$$

Name stops  $s_1, s_2$

$$\theta \in [\lambda_1(x, y, s_1),$$

$$\lambda_2(x, y, s_2)]$$

adjacency:

"crossing rules"  $\Rightarrow$  all bits in

"  $\Rightarrow$  intervals of two cells must share  $\beta$  over overlapping intervals at boundary  $\beta$

Example

~~BE~~

Type 1 boundary:  $\beta$   
line seg || edge  $E$

cell  $(R, (s_1, s_2))$  is adjacent to cell  $(R', (s'_1, s'_2))$

$$\text{iff } [s'_1, s'_2] = [s_1, s_2]$$

or

$$[s'_1, s'_2] = [s_1, E]$$

or

$$[s'_1, s'_2] = [E, s_2] \dots$$

are precise rules in lattice



## 2 Translation and Rotation in the Plane

2

- One element in one pair in  $\sigma(x, y)$  changes.

If  $\beta$  is a redundant section of a critical curve, then  $\sigma(x, y)$  is unchanged when  $\beta$  is crossed.

Thus, the crossing rules for the different types of critical curves can be generalized in the single following rule, which is valid for any critical curve section  $\beta$ , if no two critical curves coincide along  $\beta$ :

Connect  $cell(R, s_1, s_2)$  to  $cell(R', s_1, s_2)$  for each  $[s_1, s_2] \in \sigma(R) \cap \sigma(R')$  and connect each  $cell(R, s_1, s_2), [s_1, s_2] \in \sigma(R) \setminus \sigma(R')$ , if any, to each  $cell(R', s'_1, s'_2), [s'_1, s'_2] \in \sigma(R') \setminus \sigma(R)$ , if any.

Now, we can define and build the connectivity graph:

**DEFINITION 5:** The connectivity graph  $G$  is the non-directed graph whose nodes are all the cells  $cell(R, s_1, s_2)$ , where  $R$  is a non-

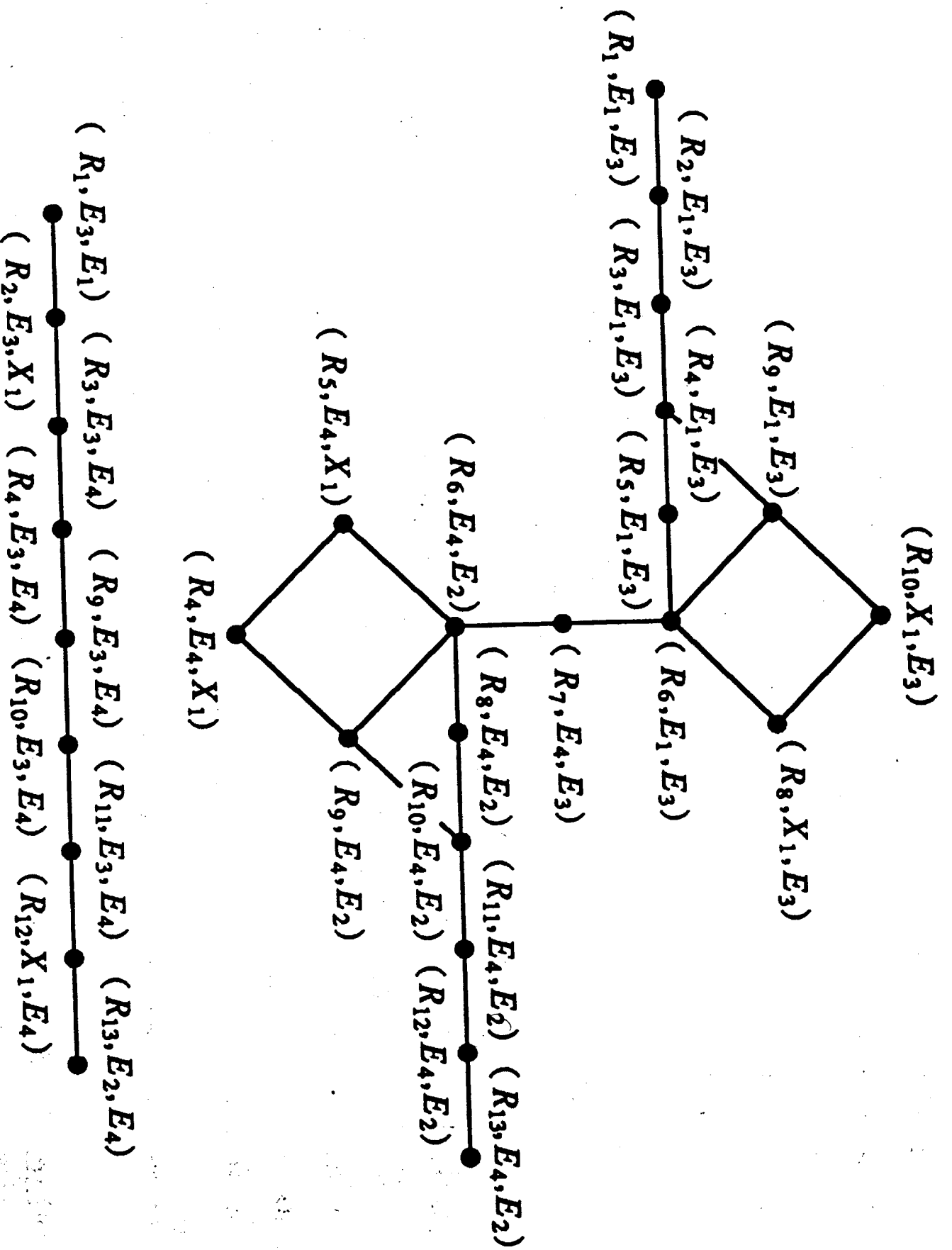


Figure 13. This figure shows the connectivity graph for the example. The graph consists of two connected components, hence verifying the lemma.

